Incoherent solitons in instantaneous nonlocal nonlinear media

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We predict random-phase spatial solitons in instantaneous nonlocal nonlinear media. The key mechanism responsible for self-trapping of such incoherent wave packets is played by the nonlocal (rather than the traditional noninstantaneous) nature of the nonlinearity. This kind of incoherent soliton has profoundly different features than other incoherent solitons.

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The observation of spatially incoherent optical solitons [1] has opened up a new direction in nonlinear science [1–8]. Such spatially incoherent solitons—self-trapped entities whose structure varies randomly in time—form when diffraction, governed by the beam’s correlation function, is robustly balanced by nonlinear self-focusing. This balance results in the stationary propagation of the beam’s envelope (i.e., its time-averaged intensity) [3]. A prerequisite for the formation of spatially incoherent solitons is that the nonlinearity responds to the envelope of the beam rather than to the fluctuating intensity pattern. Otherwise, the speckled nature of the field would induce intricate spatial variations in the refractive index, causing beam fragmentation and prohibiting self-trapping. In their original concept, incoherent solitons were studied only in rather slow nonlinear media [1–5], suppressing azimuthal instability [18,19], and it has been believed that a noninstantaneous response is a mandatory condition for the formation of such solitons [3,6]. Here, we show that if the nonlinearity has a local nature, it can filter out the otherwise highly fragmented variations in the refractive index induced by the rapidly fluctuating multimode field. In this fashion, incoherent (random-phase) spatial solitons can form in instantaneous nonlocal nonlinear media.

Nonlocal nonlinearities are inherent to many systems, when the underlying mechanism involves transport (of heat [9], atoms in a gas [10], charge carriers in semiconductors [11], etc.), long-range forces (e.g., electrostatic interactions in liquid crystals [12]), or photon attraction [13]. Nonlocality also affects the propagation of waves in plasma [14], and matter waves in Bose-Einstein condensates, where nonlocality arises from the underlying many-body interactions [15]. For localized wave packets, of which solitons are an exemplary phenomenon, nonlocality becomes important when the range of nonlocal interactions is appreciable on the scale of the variations in the beam profile. Nonlocality has profound consequences on solitons [16–20] by arresting catastrophic collapse [17], suppressing azimuthal instability [18,19], and giving rise to attractions between dark solitons (that otherwise repel) [20], etc.

Here, we predict a new type of incoherent soliton, forming in a fast nonlocal nonlinear medium, i.e., a medium responding much faster than the characteristic fluctuation time \( t_c \). Such “instantaneous” incoherent solitons form when (i) the beam is self-trapped within a time frame much shorter than \( t_c \), and (ii) the transverse momentum of the beam is constant in time. When the latter condition is violated, the time-averaged intensity of the beam exhibits a new type of a propagation-broadening mechanism: Statistical nonlinear diffraction.

The propagation of weakly correlated waves in the fast-responding nonlocal nonlinear media is an issue of generic interest. Here, we analyze this problem in the context of optics. Consider a quasimonochromatic partially incoherent beam, propagating in a fast-responding nonlocal nonlinear medium. The characteristic speckle size (=transverse correlation distance) is at least several times larger than the wavelength; hence the paraxial approximation is valid. The characteristic time scales involved are \( \tau, t_c \), and the “time of flight” \( t_f \) during which a light beam passes through the medium. Here, we consider the regime in which \( \tau \ll t_c, t_f \). From the sources typically used to study random-phase solitons (e.g., rotating diffusers [1]), the coherence time is a controllable variable. Hence, the relations above and the nonlinear dynamics we suggest below are experimentally accessible in all nonlocal nonlinearities. This said, especially attractive are very fast highly nonlocal nonlinearities that were so far believed to be inaccessible for supporting random-phase solitons, simply because their response would not average out the random intensity fluctuations. For example, with the ideas presented here, random-phase solitons can be generated in semiconductors [11], in which the self-focusing on/off response time is associated with the recombinations time of the charge carriers (typically subnanosec), and is also highly nonlocal due to the charge carriers high mobility [11].

The complex field \( \Psi(x,z,t) \) describing the spatially incoherent light is fluctuating with a characteristic time scale \( t_c \). We study the propagation of \( \Psi(x,z,t) \) in two steps. First, we analyze the propagation within a very short time interval \( \ll t_f \) during which the beam can be treated as a coherent speckled (multimode) wave. Second, we calculate the propagation of the time-averaged envelope [21]. Assuming that the incoherent light source is ergodic, the time average corresponds to an ensemble average over possible realizations of the speckled field. In addition, because the response time of the nonlinear medium is very short, the nonlinearity has no memory, and we resort to ensemble averaging while analyz-
ing the propagation of the (time-averaged) beam envelope.

First we study the dynamics within the short time frame. The dimensionless, slowly varying amplitude of the optical field $\Psi(x,z,t)$ evolves according to [22]

$$\frac{\partial \Psi}{\partial t} + \nabla^2 \Psi + \Delta n(\Psi) \Psi(x,z,t) = 0,$$

where $x$ and $z$ are the dimensionless transverse and propagation directions, respectively. The nonlocal nonlinear term $\Delta n$ has the form of a spatial convolution between the instantaneous wave intensity $|\Psi(x,z,t)|^2$ and the response function $R(x',x') = R(x-x')$ of the medium [17]

$$\Delta n(x,z,t) = \int_{-\infty}^{\infty} R(x'-x)|\Psi(x',z,t)|^2 dx'.$$

For concreteness, consider a Gaussian response function $R = 1/(\pi \sigma^2) \exp[-(x-x')^2/\sigma^2]$. Here, we are interested in the highly nonlocal regime, i.e., the width of the response function $\sigma$ is much larger than the width of the beam. In this regime, the nonlinear index change averages over the variations in the beam profile, and $\Delta n$ has an approximately parabolic shape, depending only on the total power [16,17]

$$\Delta n(x,z,t) \approx \frac{P(t)}{\sqrt{\pi \sigma}} \left[ 1 - \left| \frac{x-a(z,t)}{\sigma} \right|^2 \right],$$

where $P(t) = \int_{-\infty}^{\infty} |\Psi(x,z,t)|^2 dx$ is the total power of the beam within this time frame, and $a(z,t) = \int_{-\infty}^{\infty} |\Psi(x,z,t)|^2 dx$ is the “beam center” at plane $z$. The center of the induced waveguide coincides with the center of the beam $a(z,t)$. The beam enters the nonlocal medium at angle $\theta(t)$ (with respect to $z$) which varies stochastically with time. This propagation angle corresponds to the initial transverse momentum of the beam: $\theta(t) = 2\vec{k}_\perp(t) = 2\int_{-\infty}^{\infty} \vec{k}_\perp(\vec{k}_\perp, z=0, t)^2 d\vec{k}_\perp$ (the factor 2 appears because in our dimensionless units, $\vec{k}_\perp = \vec{z}$). Because Eq. (1) conserves transverse momentum, the angle does not change along $z$, and the center of the waveguide $a(z,t)$ lies on a straight line: $a(z,t) = a(0,0) + 2\vec{k}_\perp(t)z$.

The discussion above assumes that the width of the beam is much smaller than the nonlocality range $\sigma$ for all $z$. Let us examine when this happens. Consider a beam $\Psi(x,z=0,t)$ that at $z=0$ is much narrower than the nonlocality range $\sigma$. In this limit, the beam induces a parabolic waveguide at the vicinity of the input face, and some of its guided modes are excited by $\Psi(x,z=0,t)$. If all the modes excited by the beam are much narrower than $\sigma$, the highly nonlocal limit is satisfied throughout the propagation. In this situation, the instantaneous induced waveguide is stationary with a straight line trajectory of $a(z,t)$, while the beam is populating its guided modes in a self-consistent fashion [23]. The instantaneous beam is thus self-trapped, yet its intensity oscillates periodical due to “beating” among the modes comprising it.

Interestingly, having a self-trapped speckled beam at any instantaneous time frame does not necessarily guarantee that the time-averaged behavior of such a beam exhibits self-trapping. This is because the initial propagation angle of the beam, $\theta(t)$, and its transverse displacement $a(0,0)$, fluctuate randomly on time-scale $t_c$. We, therefore, examine the time-average behavior of the system, with the average taken over $t \gg t_c$. From ergodicity, such averaging is equal to an ensemble average over all possible initial conditions $\theta(0)$ and $a(0,t)$. Let us denote $p(\vec{k}_\perp)$ as the probability distribution of the transverse momentum of the incident beam. Consider first the case where $\vec{k}_\perp(t)$ is a random variable, hence $p(\vec{k}_\perp)$ has some width. When self-consistency is satisfied (the beam is self-trapped in each frame), the instantaneous intensity at a large enough $z$ is located in the vicinity of the beam center, $a(z,t) = 2\vec{k}_\perp(t)z$. Thus, the time-averaged intensity $\langle |\Psi(x,z,t)|^2 \rangle$ after a large distance $z$ assumes the shape of $p(\vec{k}_\perp)z$. That is, the time-averaged intensity broadens, with a width proportional to $z$ and to the width of the probability distribution of the transverse momentum of the light (defined by the source) $p(\vec{k}_\perp)z$. Consequently, an incoherent beam in a nonlocal nonlinear medium may form self-trapped solitonic beams in each short time frame, while its time-averaged intensity structure is broadening. We emphasize that this propagation broadening is nonlinear, arising because an incoherent source typically emits light with stochastically varying directionality. Henceforth we address this new propagation-broadening mechanism as a statistical nonlinear diffraction.

There are cases, however, when the statistical nonlinear diffraction is eliminated. Such cases occur, for example, when the source emitting the incoherent light does not have randomly fluctuating transverse momentum, i.e., when $p(\vec{k}_\perp) = \delta(\vec{k}_\perp)$, thus forming an instantaneous self-trapped beam with $\theta = 0$ at all times $t$. In this case, the beam self-traps within each short time frame and also forms a time-averaged random-phase soliton.

Let us now analyze some examples. Consider first a beam which at $z=0$ is a superposition of two uncorrelated coherent Gauss-Hermite waves [24]

$$\Psi(x,z=0) = \frac{\exp(-x^2)}{2\sqrt{\pi}} \left[ \sqrt{\frac{P_m}{2\pi n^2}} H_n(\sqrt{2}x) + \sqrt{\frac{P_m}{2\pi m^2 n^2}} H_m(\sqrt{2}x) \exp(i\phi(t)) \right],$$

where $n \neq m$, $H_n$ is the Hermite polynomial of order $n$, $P_m$ is the modal power of wave $n$, and $\phi(t)$ is a real random variable uniformly distributed in the interval $[-\pi, \pi]$. Such a beam, with $P_m$ and $P_n$ being constants and $\phi(t)$ stochastic, has been used in the past to generate multimode solitons [25]. In the highly nonlocal limit, this beam induces a parabolic waveguide whose width depends only on the total power $P = P_m + P_n$. First, we set $P_m = P_n = 2\sqrt{\pi} \sigma^3$. For such $P$ values, the uncorrelated waves of $\Psi(x,z=0,t)$ coincide with the guided (Gauss-Hermite) modes of the induced waveguide. To work out the time-averaged propagation of such a beam in our system, we average over the propagation of 100 realizations $(\theta_j = \pi/2, j = 0, 1, 2, 3, \ldots, 100)$. In each frame, we simulate the evolution of the coherent beam, $\Psi_j$, [via Eq. (1)], assuming a Gaussian response function [via Eq. (2)] with $\sigma = 20$.

Figure 1 presents the results with $n=0$ and $m=1$. Figures 1(a) and 1(b) show two representative frames: $\phi = 0$ [Fig. 1(a)] and $\phi = \pi/2$ [Fig. 1(b)]. Within each frame, the beam is self-trapped, yet the instantaneous beams are propagating...
with a fast fluctuating directionality. Consequently, the time-
(ensemble-) averaged beam [Fig. 1(c)] broadens due to sta-
tistical nonlinear diffraction. For comparison, we simulate
the linear propagation of the time-averaged intensity [Fig.
1(d)]. After some distance [shown in Fig. 1(e) for \( z = 3 \)],
the statistical nonlinear diffraction (dashed) leads to a differ-
tent beam profile compared to the profile of the linearly diffrac-
ting beam (solid), although the two intensity maxima in both
cases coincide. The spatial power spectrum and the prob-
bility distribution of the transverse momentum are plotted in
Fig. 1(f). The calculated profiles [Fig. 1(f)] resemble the cal-
culated linear and nonlinear diffraction profiles [Fig. 1(e)].
This shows that the far-field time-averaged intensity struc-
ture of the statistical nonlinear diffraction indeed corre-
sponds to the probability distribution of the transverse mo-
nentum of the incident beam.

An incoherent soliton forms in our system if the statistical
nonlinear diffraction is eliminated, i.e., \( p(\vec{k}) = \delta(\vec{k}) \), which
for a wave given by Eq. (4) occurs whenever \( n \neq m \pm 1 \). Figure
2 shows an example of a soliton comprising of modes 0
and 2. Figures 2(a) and 2(b) show two representative frames:
\( \varphi = 0 \) [Fig. 2(a)] and \( \varphi = \pi/2 \) [Fig. 2(b)]. Now, in every
time frame, the beam not only self-traps, but is also always prop-
gating exactly on axis. The ensemble average is shown in
Fig. 2(c), demonstrating the stationary propagation of the
time-averaged envelope. Figure 2(c) is a representative mul-
timode soliton occurring when the uncorrelated coherent
waves of the incident light coincide with the guided modes of
the induced waveguide. When the guided modes do not
exactly coincide with the uncorrelated coherent waves of the
incident light, the waveguide modes are excited with some correlation among them. Consequently, “beating” among the
(partially correlated) guided modes will now lead to oscilla-
tions of the envelope along propagation. Such an example is
shown in Fig. 2(d), where the modal powers of the incident
beam are 20% smaller than those of Fig. 2(c). This results in a
20% shallower induced waveguide, supporting guided modes with a different structure than the structure of the incident beam. Next, we study the case where the modal powers fluctuate randomly. In this case we calculate the
propagation of the time-averaged beam via the Monte Carlo
method. Figure 2(e) shows the propagation of a beam in
which each modal power has a Gaussian distribution with an
average \( \langle P_n \rangle = \frac{\langle P_n \rangle}{\Delta n = 0.2 \langle P_n \rangle} \) [values of Fig. 2(c)] and
standard deviation \( \sigma_n = 0.2 \langle P_n \rangle \). As shown there, the beam self-
traps into an incosherent “soliton breather”. For comparison,
Fig. 2(f) shows the linear diffraction of the time-averaged beams.

As a last example, consider a quasithermal light beam
propagating in our system. Obviously, quasithermal light ex-
cites consecutive modes of the induced waveguide, hence
statistical nonlinear diffraction is always present. However,
for highly incoherent beams, the number of excited modes is
very large and hence the ratio of consecutive pairs to incon-
secutive pairs can be very small. In this case the contribution
of statistical nonlinear diffraction to the propagation of the
time-averaged beam is very small. For example, consider a
beam consisting of 50 coherent waves

\[
\Psi(x,t,z=0) = \frac{\exp(-x^2)}{\sqrt{2\pi}} \sum_{m=0}^{n=49} \sqrt{\frac{P_m(t)}{2^{n_m}}} H_n(\sqrt{2}x) \exp[i\varphi_n(t)]
\]

incident upon the nonlinear medium. \( \varphi_n \) are statistically independent random variables. The modal powers \( P_n(t) \) are
random variables having a Gaussian distribution with \( \langle P_n \rangle = P \exp(-n/\Delta) / \sum_{n=0}^{m=49} \exp(-n/\Delta) \). \( \Delta = 25 \), \( P = 4 \sqrt{\pi} \sigma^2 \), and \( \sigma_n = 0.2 \langle P_n \rangle \). Figures 3(a) and 3(b) show the propagation of two realizations of the incident beam. In each frame the beam is
self-trapped, yet the beam in different frames is propagating in
different directions, producing the statistical nonlinear dif-
frac tion of the time-averaged envelope [Fig. 3(c)]. However,
since the beam has 50 modes, the statistical nonlinear diffraction is comparable to the linear diffraction [Fig. 1(e)] implies that the ratio between the linear and nonlinear broadening increases as the number of modes increases. That is, for the same value of nonlinearity, the more incoherent the beam, the more stationary its time-averaged intensity. There is no “penalty” for making the self-trapped beam more incoherent in a highly nonlocal nonlinear medium; in fact, lower spatial coherence results in “better” self-trapping (as long as the self-consistency is satisfied). Finally, the power spectrum and the probability distribution of the transverse momentum highlight the similarity between the spectral profiles [Fig. 3(f)] and the calculated diffraction profiles [Fig. 3(e)].

In conclusion, we have studied the propagation of spatially incoherent beams in a fast-responding (instantaneous) nonlocal nonlinear medium. A soliton forms in this system when (i) the beam is self-trapped within a time frame much shorter than the characteristic fluctuation time, and (ii) the transverse momentum of the incident beam is constant in time. When the transverse momentum randomly fluctuates in time, the beam exhibits a new kind of diffraction broadening denoted as statistical nonlinear diffraction.

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[6] Another type of incoherent soliton was discovered by A. Picozzi and M. Haelterman, Phys. Rev. Lett. 86, 2010 (2001); A. Picozzi, M. Haelterman, S. Pitois, and G. Millot, ibid. 92, 143906 (2004). These solitons exist in local instantaneous nonlinearities, forming by virtue of parametric energy exchanges between several waves. The “parametric incoherent solitons” are fundamentally different from those we describe in this Communication.
[21] The short time frame corresponds to monitoring the instantaneous intensity, while the long time frame corresponds to monitoring the intensity with a slow (relative to t_c) camera. Both times are defined by the source (via t_c), and are not related to the nonlinear response time.
[22] The transformation of the dimensionless Eqs. (1) and (2) into dimensional units (X,Z) is given by Z = zce/ωΔn_0 and X = xc/√2ωn_0Δn_0, where c is the speed of light in vacuum, ω is the angular frequency, n_0 is the linear index of refraction, and Δn_0 = n_2l_0, where n_2 is the Kerr coefficient and l_0 is a constant obtained by |I_q| = I(x,z,t)/|Ψ(x,z,t)|^2, where I is the beam’s instantaneous intensity.