Microscopic analysis of shape evolution and triaxiality in germanium isotopes

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1. INTRODUCTION

The great majority of nonspherical atomic nuclei display axially symmetric quadrupole deformed shapes; that is, their equilibrium shapes correspond to prolate or oblate ellipsoids. In a number of nuclei, however, axial symmetry is explicitly broken and the corresponding ellipsoid can be characterized by a certain degree of triaxiality. And while empirical evidence of stable triaxial shapes has been established in the excitation region of relatively high angular momenta [1–5], static triaxiality in equilibrium nuclear configurations still presents an open question, both experimentally and in microscopic nuclear structure theories. Static quadrupole shape deformations can be described in terms of the polar deformation parameters $\beta$ and $\gamma$ [6]. The parameter $\beta$ is proportional to the intrinsic quadrupole moment, and the angular variable $\gamma$ specifies the shape. The limit $\gamma = 0$ corresponds to axial prolate shapes, whereas the shape is oblate for $\gamma = \pi/3$. Intermediate values $0 < \gamma < \pi/3$ are associated with triaxial shapes. Most theoretical analyses of triaxiality have been based on two simple elementary models: (i) the rigid-triaxial rotor model of Davydov and Filippov (DF) [7], and (ii) the $\gamma$-unstable rotor model of Wilets and Jean (WJ) [8]. The assumption of the DF model is that the collective potential has a stable minimum at a particular value of $\gamma$ ($\gamma$-rigid potential), whereas in the WJ model the potential is independent of $\gamma$ and thus the corresponding collective wave functions are extended in the $\gamma$ direction ($\gamma$-soft potential).

The type of $\gamma$ deformation and the degree of triaxiality is difficult to identify from low-spin data because both the DF and the WJ models predict similar excitation energies and $B(E2)$ values for transitions within the ground-state band. Fortunately, $\gamma$ bands are much more sensitive to triaxial deformation and, in particular, the pattern of odd- and even-spin level staggering in the $\gamma$ band is different for $\gamma$-rigid and $\gamma$-soft triaxial shapes [9,10]. In a recent microscopic study [11] based on nuclear density functional theory (DFT), we investigated the emergence of $\gamma$ deformation for a large set of representative nonaxial medium-heavy and heavy nuclei. Starting from microscopic energy surfaces as functions of the polar deformation parameters $\beta$ and $\gamma$, we calculated and analyzed the systematics of low-lying collective spectra and transition rates. The analysis has clearly demonstrated that neither the limit of rigid-triaxial rotor (DF) nor the the $\gamma$ unstable rotor limit (WJ) are actually realized in nuclei. As a robust regularity in the low-spin excitation spectra, we found that typical nonaxial medium-heavy and heavy nuclei lie almost exactly in the middle between the two geometrical limits.

Experimental evidence for rigid triaxial deformation at low energy in $^{76}$Ge has recently been reported in Ref. [12]. The $\gamma$ band of this nucleus was extended and analyzed up to spin $9^+$. It was shown that the phase of the odd-even staggering of the $\gamma$ band is consistent with the assumption of a $\gamma$-rigid structure, although the amplitude of the staggering is considerably smaller than that predicted by the DF model. More specifically, the $E(2^+_1)/E(2^+_2)$ ratio, the phase of a staggering parameter that quantifies how adjacent levels within the $\gamma$ band are grouped, and the $B(E2)$ ratios are reproduced by the DF model, thus suggesting a shape with near maximum triaxiality $\gamma \approx 30^\circ$. The problem, however, is that the amplitude of the observed staggering is significantly smaller than the DF model prediction for $\gamma = 30^\circ$. The calculated staggering amplitude can be reduced by decreasing $\gamma$ but this increases the excitation energy of the $\gamma$ band significantly, in disagreement with experiment. In Ref. [12] the $\gamma$-band data were also compared to results of a phenomenological pairing-plus-quadrupole

Background: The motivation for this study is the experimental evidence for rigid triaxial deformation at low energy in $^{76}$Ge that was recently observed.

Purpose: Quadrupole shapes and low-energy spectra of the isotopes $^{72–82}$Ge are analyzed using a theoretical framework based on nuclear density functional theory.

Method: The relativistic functional DD-PC1, supplemented by a finite-range pairing force, is used to perform constrained triaxial mean-field calculations of energy surfaces as functions of quadrupole deformation parameters. The corresponding collective Hamiltonian, based on DD-PC1, is employed in the calculation of excitation spectra and transition rates.

Results: Model calculations reproduce the empirical trend of collective observables and predict the evolution of shapes from weakly triaxial in $^{74}$Ge to $\gamma$ soft in $^{78,80}$Ge. For $^{76}$Ge, in particular, the theoretical excitation spectrum is in good agreement with available data, the experimental ratio $E(2^+_1)/E(2^+_2)$ is reproduced, as well as the pattern and amplitude of the staggering in energy between odd- and even-spin states in the $\gamma$ band.

Conclusions: The mean-field potential of $^{76}$Ge appears to be $\gamma$ soft. Collective correlations drive the nucleus toward triaxiality but do not stabilize a rigid triaxial shape. Both the experimental and theoretical staggering of levels in the $\gamma$ band display a pattern consistent with triaxial shapes but the amplitudes are negligible and do not present evidence for rigid triaxiality.
shell-model calculation [13]. It was noted that, using single-
particle energies adjusted to reproduce the energy spectra of
low-lying states in neighboring odd-A nuclei, and adjusting
the interaction strengths to fit the energy levels of yrast
and other low-lying states of odd-A and even-even nuclei in this
mass region [13], shell-model calculations reproduce both the
phase and the magnitude of the $\gamma$-band staggering in $^{76}$Ge.

The evolution of triaxiality in Ge and Se nuclei and,
in particular, the rigid $\gamma$ deformation at low-spin in $^{76}$Ge,
has very recently been investigated in the framework of the
multiquasiparticle triaxial projected shell model (TPSM) [14].
It has been shown that to reproduce the data for both the yrast
and $\gamma$-vibrational bands of $^{76}$Ge, a fixed triaxial deformation
parameter $\gamma \approx 30^\circ$ is required for the TPSM calculation,
consistent with the prediction of the DF model. In a systematic
study of neighboring nuclei it has also been demonstrated
that configuration mixing of various quasiparticle states can result
in a dynamical change from a $\gamma$-rigid structure to $\gamma$-soft
shapes.

The purpose of the present work is to analyze the shapes
and low-energy spectra of the isotopes $^{72-82}$Ge using a theo-
retical framework based on nuclear density functional theory.
Nuclear energy density functionals (EDFs) provide an accurate
description of equilibrium mean-field properties and collective
excitations over the entire chart of nuclides. When compared
with the shell-model approach, already at the self-consistent
mean-field (SCMF) level one of the principal advantages is
the use of global effective interactions that can be applied to
all mass regions. Another strong point are model spaces that
include all occupied states (no distinction between core and
valence nucleons, no need for effective charges) and, of course,
mean-field results can be interpreted using intuitive picture of
intrinsic shapes. To compute excitation spectra and transition
rates, however, the EDF framework has to be extended to take into account the restoration of symmetries broken in the
mean-field approximation, and fluctuations in the collective
coordinates. The quadrupole collective model Hamiltonian
that will be used in this study is based on constrained
triaxial self-consistent mean-field calculations, including $\beta$
and $\gamma$ deformations. The resulting self-consistent solutions
(single-nucleon wave functions, occupation probabilities and
quasiparticle energies) that correspond to individual points
on the constrained energy surface are used to calculate
the parameters of the collective Hamiltonian: three mass
parameters, three moments of inertia, and the zero-point
energy correction [15]. The subsequent diagonalization of
the Hamiltonian yields the excitation energies and collective
wave functions that are used to calculate observables. In the
present calculation the relativistic functional DD-PC1 [16] is
used in the particle-hole channel, and a finite-range pairing
force separable in momentum space in the particle-particle
channel [17]. The semimicroscopic relativistic functional
DD-PC1 was adjusted to the experimental masses of a set of
64 deformed nuclei in the mass regions $A \approx 150-180$ and
$A \approx 230-250$, and further tested in a number of mean-field
and beyond-mean-field calculations in different mass regions [18].
The pairing interaction is completely determined by two
parameters adjusted to reproduce the empirical bell-shaped
pairing gap in symmetric nuclear matter [19].

We note that, on the self-consistent mean-field level, the
ground-state deformations of the Ge isotopes were also investi-
gated in the framework of Gogny–Hartree–Fock–Bogoliubov
(HFB) theory, and the Skyrme Hartree–Fock plus pairing in the
BCS approximation [20]. Five different Skyrme parametrizations
were used to explore the influence of different effective masses
and spin-orbit models. All the models predict the occurrence of triaxial shapes in Ge isotopes, with only few exceptions
that can be attributed to neutron subshell closures. The general softness of the Ge isotopes with respect to nonaxial
deformations was nicely illustrated by computing constrained
triaxial potential energy surfaces.

II. EVOLUTION OF SHAPES IN $^{72-82}$Ge

Our microscopic analysis of shape evolution in the chain of
$^{72-82}$Ge isotopes starts with a self-consistent relativistic
Hartree–Bogoliubov (RHB) [21,22] calculation of quadrupole
binding energy surfaces. The Dirac–Hartree–Bogoliubov
equations are solved by expanding the nucleon spinors in
the basis of a three-dimensional (3D) harmonic oscillator in
Cartesian coordinates. The map of the energy surface as a
function of quadrupole deformation is obtained by imposing
constraints on the axial and triaxial mass quadrupole moments.
The method of quadratic constraint [23] uses an unrestricted
variation of the function

$$ (\hat{H}) + \sum_{\mu=0,2} C_{2\mu} (\langle \hat{Q}_{2\mu} \rangle - q_{2\mu})^2, $$

where $\langle \hat{H} \rangle$ is the total energy, and $\langle \hat{Q}_{2\mu} \rangle$ denotes
the expectation value of the mass quadrupole operators,

$$ \hat{Q}_{20} = 2x^2 - x^2 - y^2 \quad \text{and} \quad \hat{Q}_{22} = x^2 - y^2, $$

$q_{2\mu}$ is the constrained value of the multipole moment, and $C_{2\mu}$
is the corresponding stiffness constant.

In Fig. 1 we display the RHB triaxial quadrupole energy
maps of the even-even isotopes $^{72-82}$Ge in the $\beta-\gamma$
plane ($0 \leq \gamma \leq 60^\circ$). For each nucleus energies are normalized
with respect to the binding energy of the absolute minimum.
Because of the $N = 40$ subshell closure $^{72}$Ge displays a
pronounced spherical minimum. By adding just two more neu-
trons a pronounced triaxial minimum at ($\beta, \gamma$) = (0.25,32\degree)
develops in $^{74}$Ge. Additional neutrons at first lead to a softening
of the energy surface in the $\gamma$ direction, resulting in the
axially symmetric minimum on the prolate axis for the isotopes
$^{76-80}$Ge. Finally, $^{82}$Ge is a semimagcic spherical nucleus.

The variation of mean-field shapes in an isotopic chain is
governed by the evolution of the underlying shell structure of
single-nucleon orbitals. The formation of deformed minima
is related to the occurrence of regions of low single-particle
level density around the Fermi surface. In Figs. 2–4 we plot
the proton and neutron single-particle energy levels in the
canonical basis for the nuclei $^{74,76,78}$Ge. Solid (blue) and
dashed (red) curves correspond to levels with positive and
negative parity, respectively. The dot-dashed (green) curves
denote the Fermi level. The single-particle levels are plotted
as functions of the deformation parameters along closed paths
in the $\beta-\gamma$ plane. The panels on the left and right display
prolate ($\gamma = 0^\circ$) and oblate ($\gamma = 60^\circ$) axially symmetric

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single-particle levels, respectively. In the middle panel of each figure the proton and neutron levels are plotted as functions of $\gamma$, for a fixed value of the axial deformation $|\beta|$ at the position of equilibrium minimum of the binding energy surface. This means that, starting from the spherical configuration, one follows the single-nucleon levels on a path along the prolate axis up to the approximate position of the minimum (left panel). Next, for this fixed value of $|\beta|$ we trace the levels along the path from $\gamma = 0^\circ$ to $\gamma = 60^\circ$ (middle panel) and, finally, back to the spherical configuration along the oblate axis (right panel). Configurations along the oblate axis are denoted by negative values of $\beta$. For $^{72}\text{Ge}$ one notices that the proton levels display a pronounced gap between the last occupied and first unoccupied levels in the triaxial region at $\gamma \approx 30^\circ$. For the neutron levels the gap appears to be almost independent of $\gamma$. Combined self-consistently, the proton and neutron gaps lead to the formation of the triaxial minimum on the energy surface shown in Fig. 1. This microscopic picture does not change for the proton levels of $^{76}\text{Ge}$ (Fig. 3) and $^{78}\text{Ge}$ (Fig. 4), but the neutron levels display a tendency towards
for a fixed value of the axial deformation each figure the proton and neutron levels are plotted as functions of symmetric single-nucleon levels, respectively. In the middle panel of closed paths in the left and right display prolate (\(\gamma = 0^\circ\)) and oblate (\(\gamma = 60^\circ\)) axially symmetric single-nucleon levels, respectively. In the middle panel of each figure the proton and neutron levels are plotted as functions of \(\gamma\) for a fixed value of the axial deformation \(|\beta|\) that corresponds to the mean-field minimum.

prolate axially symmetric deformation. The dependence of the energy on the triaxial deformation parameter \(\gamma\) is illustrated even more clearly with the projections shown in Fig. 5, where we plot the self-consistent RHB constrained energy curves of \(^{74}\text{Ge}, \ ^{76}\text{Ge}, \text{and} \ ^{78}\text{Ge}\) as functions of \(\gamma\), at fixed values of the axial deformation: \(\beta = 0.25, \beta = 0.20, \text{and} \ \beta = 0.20\), respectively, that correspond to the positions of the mean-field minima in Fig. 1. One notices that \(^{74}\text{Ge}\) displays a shallow triaxial minimum at \(\gamma = 30^\circ\), whereas the isotopes \(^{76}\text{Ge}\) and \(^{78}\text{Ge}\) have axially symmetric minima. The magnitude of the \(\gamma\) dependence of the energy is similar for \(^{74}\text{Ge}\) and \(^{76}\text{Ge}\), whereas it is considerably stiffer for \(^{78}\text{Ge}\). In all three cases, however, the difference in energy between the prolate and triaxial configurations is less than 1.5 MeV and, in particular, for \(^{76}\text{Ge}\) the prolate minimum is located only 0.5 MeV below the triaxial \(\gamma = 30^\circ\) configuration. Such a small energy difference between mean-field configurations characterized by different deformation parameters indicate a potentially decisive role of dynamical effects related to restoration of broken symmetries and fluctuations in collective coordinates. As shown in the global study of quadrupole correlation effects of Ref. [24], typically nuclei below mass \(A \leq 60\) display larger dynamical correlation energy than static deformation energy (energy difference between the spherical configuration and the deformed equilibrium configuration), whereas heavier deformed nuclei have larger static deformation energy than collective correlation energy.

![Figure 2](image2.png)

FIG. 2. (Color online) Single-proton and single-neutron energy levels of \(^{74}\text{Ge}\) as functions of the deformation parameters along closed paths in the \(\beta-\gamma\) plane. Solid (blue) and dashed (red) curves correspond to levels with positive and negative parity, respectively. The dot-dashed (green) curves denote the Fermi level. The panels on the left and right display prolate (\(\gamma = 0^\circ\)) and oblate (\(\gamma = 60^\circ\)) axially symmetric single-nucleon levels, respectively.

![Figure 3](image3.png)

FIG. 3. (Color online) Same as described in the caption to Fig. 2 but for the nucleus \(^{76}\text{Ge}\).

![Figure 4](image4.png)

FIG. 4. (Color online) Same as described in the caption to Fig. 2 but for the nucleus \(^{78}\text{Ge}\).

![Figure 5](image5.png)

FIG. 5. (Color online) Self-consistent RHB constrained energy curves of \(^{74}\text{Ge}, \ ^{76}\text{Ge}, \text{and} \ ^{78}\text{Ge}\) as functions of the deformation parameter \(\gamma\), at fixed values of the axial deformation: \(\beta = 0.25, \beta = 0.20, \text{and} \ \beta = 0.20\), respectively. For each nucleus energies are normalized with respect to the binding energy of the absolute minimum.
To include collective correlations and, therefore, to enable calculation of excitation spectra and transition rates, a collective Hamiltonian can be formulated that restores rotational symmetry and accounts for fluctuations around the axial or triaxial mean-field minima. The dynamics of the five-dimensional Hamiltonian for quadrupole vibrational and rotational degrees of freedom is governed by the seven functions of the intrinsic deformations $\beta$ and $\gamma$: the collective potential, the three vibrational mass parameters: $B_{\beta\beta}$, $B_{\gamma\gamma}$, and $B_{\beta\gamma}$, and three moments of inertia for rotations around the principal axes. The microscopic self-consistent solutions of the constrained triaxial RHB equations, i.e., the single-quasiparticle energies and wave functions for the entire energy surface as functions of the quadrupole deformations, provide the microscopic input for the parameters of the collective Hamiltonian [15]. The five quadrupole collective coordinates are parameterized in terms of two deformation parameters $\beta$ and $\gamma$ and three Euler angles $(\phi, \theta, \psi) \equiv \Omega$, which define the orientation of the intrinsic principal axes in the laboratory frame. The collective Hamiltonian reads

$$\hat{H} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}. \quad (3)$$

with the vibrational kinetic energy

$$\hat{T}_{\text{vib}} = -\frac{\hbar^2}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^4 B_{\gamma\gamma} \frac{\partial}{\partial \beta} - \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^3 B_{\beta\gamma} \frac{\partial}{\partial \gamma} \right\} + \frac{1}{\beta} \sin 3\gamma \left\{ - \frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \frac{\partial}{\partial \gamma} + \frac{1}{\beta} \frac{\partial}{\partial \gamma} \right\}, \quad (4)$$

and rotational kinetic energy

$$\hat{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^{3} \frac{\hbar^2}{\mathcal{I}_k} \mathcal{J}_k^2. \quad (5)$$

$V_{\text{coll}}$ is the collective potential. $\mathcal{J}_k$ denotes the components of the angular momentum in the body-fixed frame of a nucleus, and the mass parameters $B_{\beta\beta}$, $B_{\gamma\gamma}$, and $B_{\beta\gamma}$, as well as the moments of inertia $\mathcal{I}_k$, depend on the quadrupole deformation variables $\beta$ and $\gamma$:

$$\mathcal{I}_k = 4B_k \beta^2 \sin^2(\gamma - 2k\pi/3). \quad (6)$$

Two additional quantities that appear in the expression for the vibrational energy: $r = B_1 B_2 B_3$, and $w = B_{\beta\beta} B_{\gamma\gamma} - B_{\beta\gamma}^2$, determine the volume element in the collective space.

The collective potential, the mass parameters, and the moments of inertia are determined by the microscopic nuclear energy density functional and the effective interaction in the $pp$ channel. In the current implementation of the model the moments of inertia are computed using the Inglis–Belyaev formula:

$$\mathcal{I}_k = \sum_{i,j} \frac{|\langle i| \mathcal{J}_k \rangle| \Phi_i|^2}{E_i + E_j}, \quad k = 1, 2, 3, \quad (7)$$

where $k$ denotes the axis of rotation, the summation runs over proton and neutron quasiparticle states $|i\rangle = \beta_i \frac{\beta_i^*}{\mathcal{N}} \Phi_i$, and $|\Phi\rangle$ represents the quasiparticle vacuum. The mass parameters associated with the two quadrupole collective coordinates $q_0 = \langle \hat{Q}_{20} \rangle$ and $q_2 = \langle \hat{Q}_{22} \rangle$ are calculated in the cranking approximation:

$$B_{\mu\nu}(q_0, q_2) = \frac{\hbar^2}{2} \left[ M_{\mu\nu}^{(1)}(1), M_{\mu\nu}^{(2)}, M_{\mu\nu}^{(3)} \right]_{\mu\nu}, \quad (8)$$

where

$$M_{\mu\nu}(q_0, q_2) = \sum_{i,j} \frac{|\langle \Phi \rangle \hat{Q}_{20} |i\rangle \langle i| \hat{Q}_{22} |\Phi\rangle|}{(E_i + E_j)^\mu}. \quad (9)$$

Finally, the potential $V_{\text{coll}}$ in the collective Hamiltonian (3) is obtained by subtracting the zero-point energy corrections from the total energy that corresponds to the solution of constrained RHB equations, at each point on the triaxial deformation plane [15].

The Hamiltonian of Eq. (3) describes quadrupole vibrations, rotations, and the coupling of these collective modes. The diagonalization yields the excitation energies and collective wave functions:

$$\Psi_{\alpha}^{JM}(\beta, \gamma, \Omega) = \sum_{K \in \Delta J} \psi_{\alpha K}(\beta, \gamma) \Phi_{MK}^J(\Omega). \quad (10)$$

The angular part corresponds to linear combinations of Wigner functions

$$\Phi_{MK}^J(\Omega) = \sqrt{\frac{2J + 1}{16\pi^2(1 + \delta_{K0})}} [D_{MK}^J(\Omega) + (-1)^J D_{MK}^{-J}(\Omega)], \quad (11)$$

and the summation in Eq. (10) is over the allowed set of the $K$ values:

$$\Delta J = \begin{cases} 0, 2, \ldots, J & \text{for } J \text{ mod } 2 = 0 \\ 2, 4, \ldots, J - 1 & \text{for } J \text{ mod } 2 = 1. \end{cases} \quad (12)$$

By using the collective wave functions of Eq. (10), various observables can be calculated and compared to experimental results. For instance, the quadrupole $E2$ reduced transition probability:

$$B(E2; \alpha J \to \alpha' J') = \frac{1}{2J + 1} |\langle \alpha' J'| \hat{N}(E2) |\alpha J\rangle|^2, \quad (13)$$

where $\hat{N}(E2)$ is the electric quadrupole operator.

The quality with which model calculations reproduce the trend of available data is illustrated in Fig. 6 where we plot the isotopic dependence of two characteristic collective observables: the ratio $R_{4I2}$ between the excitation energies of the first $4^+$ and $2^+$ states, and the $B(E2; 2^+_1 \to 0^+_0)$ values in Weisskopf units. The theoretical values obtained by the
are given in Weisskopf units. The functional (left) compared to data [12,25] (right). The model reproduces the empirical trend and the separable pairing force, are shown in comparison to the experimental value for $^{76}\text{Ge}$. The calculated $B(E2)$ values in Weisskopf units (right panel), with mass number for the Ge isotopes.

FIG. 6. (Color online) Evolution of the ratio between the excitation energies of the first $4^+$ and $2^+$ states (left panel), and the $B(E2; 2_1^+ \rightarrow 0_1^+)$ values in Weisskopf units (right panel), with mass number for the Ge isotopes.

The staggering can be quantified by considering the following distribution of intrinsic configurations.

The level of $K$ mixing is reflected in the staggering in energy between odd- and even-spin states in the $\gamma$ band. The staggering can be quantified by considering the following

$$N_K = 6 \int_0^{\pi/3} \int_0^\infty |\psi_{\alpha,K}(\beta,\gamma)|^2 \beta^4 \sin 3\gamma |d\beta d\gamma. \quad (14)$$

The $\psi_{\alpha,K}(\beta,\gamma)$ components are defined in Eq. (10). A broader distribution of $N_K$ values in the state $|\alpha J\rangle$ provides a measure of mixing of intrinsic configurations.
differential quantity [9]:

\[
S(J) = \frac{E[J^+\nu] - 2E[(J-1)^+\nu] + E[(J-2)^+\nu]}{E[2^+\nu]}. ~ (15)
\]

\(S(J)\) measures the displacement of the \((J-1)^+\nu\) level relative to the average of its neighbors \(J^+\nu\) and \((J-2)^+\nu\), normalized to the energy of the first-excited state of the ground-state band \(2^+\nu\). The differential form of the \(S(J)\) makes it extremely sensitive to the shape of a nucleus. For an axially symmetric rotor \(S(J)\) is constant. In a study of staggering of \(\gamma\)-band energies and the transition between different structural symmetries in nuclei [10], the experimental energy staggering in \(\gamma\) bands of several isotopic chains was investigated as a signature for the \(\gamma\) dependence of the potential. For a nucleus with a deformed \(\gamma\)-soft potential, \(S(J)\) oscillates between negative values for even-spin states and positive values for odd-spin states, with the magnitude slowly increasing with spin. For a triaxial potential the level clustering in the \(\gamma\) band is opposite, and \(S(J)\) oscillates between positive values for even-spin states and negative values for odd-spin states. In this case the magnitude of \(S(J)\) increases more rapidly with spin, as compared to the \(\gamma\)-soft potential.

Figure 9 displays the calculated values for the staggering \(S(J)\) in the \(\gamma\) bands of \(^{74-80}\text{Ge}\) isotopes. The experimental values for the isotope \(^{76}\text{Ge}\) [12] are also included in the figure. The results clearly show that the phase of the theoretical \(S(J)\) for \(^{74}\text{Ge}\) is consistent with the DF picture of a \(\gamma\)-rigid triaxial shape, although the amplitude of the staggering is considerably smaller than the one predicted by the DF model. On the other hand, \(^{76}\text{Ge}\) and \(^{80}\text{Ge}\) display the opposite pattern for \(S(J)\); that is, their \(\gamma\) bands indicate soft shapes. \(^{76}\text{Ge}\) appears to be at the transition point between the triaxial \(^{74}\text{Ge}\) and the \(\gamma\)-soft heavier isotopes. For this isotope the amplitudes of \(S(4)\) and \(S(5)\) almost vanish, whereas \(S(6)\) and \(S(7)\) follow a pattern characteristic for triaxial shapes, but with considerably smaller amplitudes. This cannot be considered as a robust indication of rigid triaxiality. As already emphasized in the introduction, in Ref. [12] it was noted that an amplitude much smaller than that predicted by the DF model could be explained by assuming that the triaxial potential has a minimum at \(\gamma < 30^\circ\), but this pushes the \(\gamma\) band higher in excitation energy, in contrast to the experimental ratio \(E(2^+\nu)/E(2^+\gamma) = 2\).

The present calculation reproduces the experimental ratio \(E(2^+\nu)/E(2^+\gamma) = 2\), the phase and the amplitude of the staggering \(S(J)\) for the low-spin levels of the \(\gamma\) band of \(^{76}\text{Ge}\). States with higher angular momenta display such a pronounced level of fragmentation of different \(K\) components that it becomes impossible to identify members of the \(\gamma\) band \((K = 2)\) unequivocally. We emphasize that, in contrast to previous shell-model calculations [13,14] that provided support for rigid triaxial deformation at low energy in \(^{76}\text{Ge}\) but used single-particle energies and two-body interactions specifically tailored to spectroscopic data in this mass region, the present results have been obtained using a universal energy density functional (DD-PC1) and a pairing interaction that were not adjusted to the considered nuclei in any way. In fact, as already noted in the introduction, the parameters of the functional DD-PC1 were determined only by the empirical masses in the regions \(A \approx 150-180\) and \(A \approx 230-250\). It is therefore remarkable that, without any further adjustment, the quadrupole collective Hamiltonian based on this functional yields results that are in such good agreement with available data for \(^{76}\text{Ge}\).

III. CONCLUSION

The framework of nuclear density functional theory has been used to analyze the evolution of quadrupole shapes in the isotopes \(^{72-82}\text{Ge}\). The motivation for this study is the experimental evidence for rigid triaxial deformation at low energy in \(^{76}\text{Ge}\) that was recently reported in Ref. [12]. Employing the universal relativistic functional DD-PC1 [16], and a finite-range pairing force separable in momentum space [17], we have used the relativistic Hartree–Bogoliubov model to calculate the constrained energy surfaces of germanium isotopes as functions of the quadrupole deformation parameters \(\beta\) and \(\gamma\). The resulting single-quasiparticle energies and wave functions for the entire energy surface as functions of the quadrupole deformations determine the parameters of the collective Hamiltonian [15] that are used to compute low-energy excitation spectra and electromagnetic transition rates.

The results for the energy surfaces (Figs. 1 and 5), for the spectrum of \(^{76}\text{Ge}\) (Fig. 7), and the staggering \(S(J)\) shown in Fig. 9, illustrate the evolution of shapes from weakly triaxial in \(^{74}\text{Ge}\) to \(\gamma\) soft in \(^{78,80}\text{Ge}\) and, finally, spherical in \(^{82}\text{Ge}\). Even though our results are in very good agreement with available data for \(^{76}\text{Ge}\), both for the ratio \(E(2^+\nu)/E(2^+\gamma) = 2\) and the pattern and amplitude of the staggering \(S(J)\), they do not confirm the evidence for rigid triaxial deformation at low energy in this nucleus. In fact, the present analysis indicates that the mean-field potential of \(^{76}\text{Ge}\) is \(\gamma\) soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but they are not strong enough to stabilize a \(\gamma \approx 30^\circ\) triaxial shape. This is clearly reflected in both the experimental and calculated staggering \(S(J)\) which display a pattern consistent with triaxial shapes but the amplitudes are negligible and cannot be considered as evidence for rigid triaxiality.